Circular Motion

Rotating Objects

- When an object rotates, all the points along the radius move through the same angle in the same amount of time.
- Therefore, the linear velocity will be different depending on the distance from the center (radius).

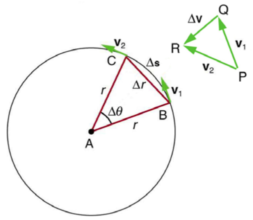
Ferris wheel animated icon created by Freepik - Flaticon

OpenStax, Rice University (CC BY 4.0)

Uniform Circular Motion

- An object that moves in a circle at constant tangential (linear) speed v, is said to experience **uniform circular motion**.
- The speed may be constant but the direction is changing.
- This means that the object is accelerating.
- The acceleration is in the direction of the change in velocity.

- The direction of the acceleration is towards the center of the circular path.



- This acceleration is called **centripetal acceleration** (towards the center or center seeking).

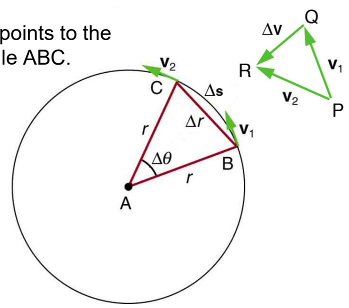
Deriving an Equation for Centripetal Acceleration

- During Δt the object moves from B to C
  - Connecting these points to the center gives triangle ABC.

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = v_2 - v_1$$

- The vector subtraction gives triangle PQR.
  - $v_1 = v_2 = v$



- The two triangles are similar triangles, therefore

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

- Solving for Δv

$$\Delta v = \frac{v \Delta r}{r}$$

- Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta r}{r \Delta t}$$

$$\frac{\Delta v}{\Delta t} = a \qquad \frac{\Delta r}{\Delta t} = v$$

$$\boxed{a_c = \frac{v^2}{r}}$$

Period and Frequency

- The concepts of period and frequency are often used with circular motion.
- The period, T, is the time required for one rotation.
- The frequency, f, is the number of rotations per second.

$$T = \frac{1}{f}$$

- It is also useful to express centripetal acceleration in terms of angular velocity, period and frequency.

Substituting $v = \frac{2\pi r}{T}$ into the expression for centripetal acceleration

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substituting $f = \frac{1}{T}$

$$a_c = 4\pi^2 r f^2$$

Example 1

- A car drives around a curve of radius 500.0 m at a speed of 25 m/s. Calculate the centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(25)^2}{500}$$

$$a_c = 1.25 \text{ m/s}^2$$

Example 2

- A 150 g ball at the end of a string is revolving in a horizontal circle of radius 0.60 m. The ball makes 2 revolutions in one second. Calculate the centripetal acceleration of the ball.

$$a_c = \frac{v^2}{r} = \frac{(2\pi r)^2}{rT^2} = 4\pi^2 r f^2$$

$$a_c = 4\pi^2(0.6)(2)^2 \ = 95 \text{ m/s}^2$$

Centripetal Force

- Any force or combination of forces can **cause** a centripetal or radial acceleration.
- Any net force **causing** uniform circular motion is called a centripetal force.
- The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration.

- According to Newton's second law of motion, $F_{net} = ma$
- For uniform circular motion, the acceleration is the centripetal acceleration $(a = a_c)$.
- Thus, the magnitude of centripetal force is

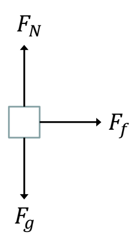
$$F_c = ma_c$$

or

$$\boxed{F_c = \frac{mv^2}{r}}$$

Example

- A 1200 kg car travels around a 500.0 m radius unbanked curve at 25.0 m/s. Calculate the minimum static coefficient of friction between the tires and the road required to keep the car from slipping.



$$F_{net} = ma$$

Since the car is moving in a circle

$$F_{net} = F_c = ma_c$$

$$F_f = ma_c$$

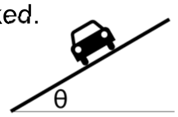
$$\mu F_N = \mu F_g = \mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{gr}$$

$$\mu = \frac{(25)^2}{(9.8)(500)} = 0.13$$

Banked Curves

- Highway and racetrack curves are not flat, they are banked.

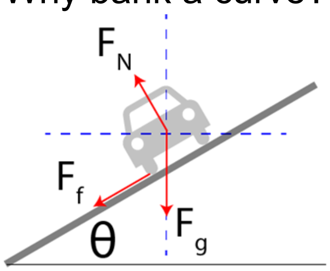


- Highway curves are typically banked a maximum of 3.5° - 4.5°.
- NASCAR race track curves vary:
  - Talladega Superspeedway - 33°
  - Daytona International Speedway - 31°
  - Indianapolis Motor Speedway - 9°

Car: tagawa (OpenClipArt)

Velocrome – khaleel (Pexels)
Car – mibro (Pixabay)
Talladega Panorama – Brian Cantoni (CC BY 2.0)

Why bank a curve?



- Friction and the normal force contribute to the centripetal force, allowing higher speeds around the curve.

Type of Curve	Maximum Speed in Curve
Flat with friction	$\sqrt{gr\mu}$
Banked without friction	$\sqrt{gr\tan\theta}$
Banked with friction	$\sqrt{gr\left(\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\right)}$